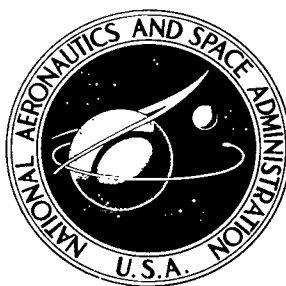


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by Carl F. Monnin and George M. Prok

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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ABSTRACT

A Maxwellian electron gas interacts with a cold neutral hydrogen gas with various degrees of dissociation. The electron temperature range is 3 to 100 eV. The plasma is assumed to be tenuous and in the steady state. The power is assumed to be lost by radiation, ionization, and kinetic energy carried to the walls per second by the charged particles. The wall loss term used is calculated from sheath theory. The cost of producing a proton in various mixtures of neutral atomic and molecular hydrogen is also presented.

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SUMMARY

A Maxwellian electron gas interacts with a cold neutral hydrogen gas with various degrees of dissociation. The electron temperature range is 3 to 100 electron volts. The plasma is assumed to be tenuous and in the steady state. The power is assumed to be lost by radiation, ionization, and kinetic energy carried to the walls per second by the charged particles. The rate at which ions arrive at the walls is assumed to equal the rate at which they are produced within the plasma. When available, experimental cross sections were used for the radiation and ionization loss terms. Otherwise, cross sections calculated from Gryzinski's semiclassical theory were used. The wall loss term used is calculated from sheath theory. Each loss term is presented as a percentage of the total power loss. It was found that radiation power loss dominates the loss mechanisms for Maxwellian temperatures below 7 electron volts and that the wall loss term dominates above 5 electron volts.

The cost of producing a proton in various mixtures of neutral atomic and molecular hydrogen is also presented. It is found that a small amount of dissociation (5 percent) reduces the proton production cost by an order of magnitude over that in pure molecular hydrogen. At 25-percent dissociation it takes only three times as much energy to produce a proton as in pure atomic hydrogen.

INTRODUCTION

In plasma research it is useful to know how much energy is lost by various loss mechanisms. In this report various energy loss mechanisms are studied for the case of a steady-state hydrogen plasma that has a Maxwellian distribution of electron energies. These results are applicable only to low electron density, optically thin plasmas with an elevated electron temperature (so called tenuous plasmas). The loss mechanisms considered are radiation, ionization, and charged particle losses to the walls.

The only radiative power loss considered is radiation from the excited states of the

hydrogen atom and molecule. This term and the ionization loss term can be easily calculated if the excitation and ionization cross sections are known. Excitation cross sections for the atom have been determined theoretically (ref. 1) and, for a few of the major transitions, experimentally (ref. 2). In addition, a modification of Gryzinski's semiclassical theory has made available the cross sections of molecular hydrogen (ref. 3). Experimentally obtained values of the cross section for ionization of atomic and molecular hydrogen are also available (ref. 2). By using these cross sections and averaging over the Maxwellian distribution function one can obtain the excitation and ionization coefficients needed to calculate the radiative and ionization energy losses in the plasma.

The energy loss to the walls is calculated by assuming that all the charged particles that strike the wall recombine and transmit all their energy to the walls. The total ion arrival rate at the wall is assumed to equal the volume ion production rate within the plasma. Assuming normal ambipolar diffusion permits determination of the strength of the potential difference across the sheath and consequently the energy added to the ions as they traverse it. It is then possible to determine the power loss to the wall as a function of electron temperature.

All of the loss mechanisms are known functions of electron temperature; thus, the effect of the degree of dissociation of molecular hydrogen can be determined for the various loss mechanisms.

The cross sections required for calculation of the proton production cost are the same as those used in radiation and ionization loss calculations. Thus, the energy cost of producing a proton in atomic and molecular hydrogen is calculated. In addition, the effect of dissociation on the proton production cost is determined.

SYMBOLS

A	dummy variable
E	energy, eV
e	electronic charge, C
f	distribution function
k	Boltzmann constant, erg/deg
m	mass, g
N	number of particles
P	power per unit volume, $\text{eV}/(\text{cm}^3)(\text{sec})$
T	temperature, K

V	potential, V
v	speed, cm/sec
\bar{v}	average speed, cm/sec
X	number fraction of atoms
σ	cross section, cm ²

Subscripts:

e	electron
H	hydrogen atom
H^+	hydrogen atomic ion
H_2	hydrogen molecule
H_2^+	hydrogen molecular ion
i	ion
ion	ionization loss
j, l	dummy subscripts
o	value in electrically neutral region
rad	radiation loss
recomb	recombination loss
v	speed
wall	wall loss
x, y, z	Cartesian coordinates
α, β	dummy subscripts

ASSUMPTIONS

The model assumes that a hot Maxwellian electron gas (3 to 100 eV) interacts with a cold neutral hydrogen gas. The hydrogen gas includes atomic and molecular hydrogen in the ground state. The plasma composition does not change. Elastic collisions are neglected in the energy range considered because the energy transfer is so small (fractional energy lost is $\sim 10^{-3}$). The ions are assumed to be cold, and therefore, charge exchange is neglected. The plasma is optically thin and partially ionized. The only energy loss mechanisms are either volume losses or wall losses. The volume loss mechanisms

are radiation and ionization. The wall losses are due to kinetic energy being carried to the wall by charged particles.

The particles carrying energy to the walls are assumed to obey sheath theory. The plasma is assumed to be uniform and cylindrical with no temperature or density gradients. End effects are neglected. The effect of a magnetic field was not considered.

THEORY

Ion Cost and Volume Loss

The model used to calculate ion production costs assumes that the only volume loss mechanism present is inelastic electron collisions. These collisions result in ionization and the radiation given off when the excited particles decay to the ground state. The plasma composition does not change and all the electron-neutral collisions occur with atomic and molecular hydrogen in the ground state. The results presented are for tenuous plasmas with low pressures ($N_0 \leq 10^{12}$ to 10^{14} cm^{-3} , $N_e \leq 10^{11}$ to 10^{12} cm^{-3}). For the case of a Maxwellian electron gas interacting with a cold neutral gas, ion production cost may be calculated from the equation (ref. 4)

$$\phi = \frac{P_{\text{ion}} + P_{\text{rad}}}{N_{\text{ion}}} \quad (\text{eV/ion}) \quad (1)$$

where

$$P_{\text{ion}} = N_0 N_e \left[X \langle \sigma_{H^+} v_e \rangle H_{H^+}^E + (1 - X) \langle \sigma_{H^+} v_e \rangle H_{H_2}^E + (1 - X) \langle \sigma_{H_2^+} v_e \rangle H_{H_2}^E \right] \left(\text{eV}/(\text{cm}^3)(\text{sec}) \right) \quad (2)$$

$$P_{\text{rad}} = N_0 N_e \left[X \sum_{\alpha} \langle \sigma_{\alpha} v_e \rangle H_{\alpha}^E + (1 - X) \sum_{\beta} \langle \sigma_{\beta} v_e \rangle H_{\beta}^E \right] \left(\text{eV}/(\text{cm}^3)(\text{sec}) \right) \quad (3)$$

$$\dot{N}_{\text{ion}} = N_0 N_e \left[X \langle \sigma_{H^+} v_e \rangle_H + (1 - X) \langle \sigma_{H^+} v_e \rangle_{H_2} + (1 - X) \langle \sigma_{H_2^+} v_e \rangle_{H_2} \right] \quad \left(1/(\text{cm}^3)(\text{sec}) \right) \quad (4)$$

The symbols P_{ion} and P_{rad} are the power per unit volume expended in the plasma by ionization and radiation processes, respectively. Equation (1) is simply the total power lost per second in a unit volume divided by the number of ions produced per second in that volume. Thus, one obtains the average energy lost per ion formed. In the previous equations N_0 and N_e are the number densities of neutral particles and electrons, respectively. The cross section and energy level of the transition are σ and E . The subscripts H and H_2 following the brackets refer to the parent gas. The subscripts α and β refer to summation over the atomic and molecular transitions, respectively. The electron velocity is v_e , and the brackets indicate an average over a Maxwellian velocity distribution. The number fraction of atomic hydrogen is

$$X = \frac{N_H}{N_H + N_{H_2}} = \frac{N_H}{N_0} \quad (5)$$

where N_H and N_{H_2} are the number densities of atoms and molecules, respectively.

The electron temperature range is 3 to 100 electron volts.

Collisions with the metastable $2s$ state of atomic hydrogen are neglected. Although the lifetime of the metastable state of atomic hydrogen is long (2.4 msec), it is easily quenched by electric fields (ref. 5).

The volume loss terms are just P_{ion} (eq. (2)) and P_{rad} (eq. (3)). The number fraction X is varied from $X = 0$ to $X = 1$ to find the effect of dissociation on these volume loss terms. The term P_{ion} includes the production of the atomic ion directly from the molecule, and P_{rad} includes all radiation from both the atom and molecule. The individual atomic energy levels were used up to $n = 3$; above this energy level a composite energy level was used to generate a composite Gryzinski cross section.

Wall Loss

It was assumed in the model that all recombination takes place at the wall, that end effects are negligible, and that the sheath thickness is very small compared to the plasma radius. Under these conditions the total number of ions produced in the plasma equals the number lost to the wall.

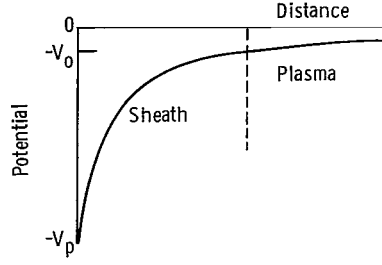


Figure 1. - Potential as function of distance from the wall.

The ions find themselves in electric fields that accelerate them to the wall (see fig. 1). Many of the electrons are repelled by these same fields. Only electrons with sufficient energy reach the wall. Thus, the number of electrons reaching the wall depends on an energy distribution. A Maxwellian distribution is assumed here. The ions must arrive at the edge of the sheath with an average energy of at least $1/2 kT_e$ (ref. 6) or a stable sheath will not form. Usually in the literature the average energy is taken to be $1/2 kT_e$.

Since we are in steady state and no volume recombination takes place, the number of H^+ ions produced equals the number lost to the walls:

$$N_o N_e \left[X \langle \sigma_{H^+ v_e} \rangle_H + (1 - X) \langle \sigma_{H^+ v_e} \rangle_{H_2} \right] \pi r^2 h = N_{H^+} \sqrt{\frac{kT_e}{m_{H^+}}} 2\pi r h \quad (1/\text{sec}) \quad (6)$$

Solving for N_{H^+} gives

$$N_{H^+} = \frac{N_o N_e \left[X \langle \sigma_{H^+ v_e} \rangle_H + (1 - X) \langle \sigma_{H^+ v_e} \rangle_{H_2} \right] r}{2 \sqrt{\frac{kT_e}{m_{H^+}}}} \quad (1/\text{cm}^3) \quad (7)$$

Also, the number of H_2^+ ions produced equals the number lost to the wall:

$$N_o N_e (1 - X) \langle \sigma_{H_2^+ v_e} \rangle_{H_2} \pi r^2 h = N_{H_2^+} \sqrt{\frac{kT_e}{m_{H_2^+}}} 2\pi r h \quad (1/\text{sec}) \quad (8)$$

which yields

$$N_{H_2^+} = \frac{N_o N_e (1 - X) \langle \sigma_{H_2^+}^{v_e} \rangle_{H_2} r}{2 \sqrt{\frac{kT_e}{m_{H_2^+}}}} \quad (1/\text{cm}^3) \quad (9)$$

The ion current must equal the electron current at the wall. Thus, the particle current equation at the wall is

$$N_{H^+} \sqrt{\frac{kT_e}{m_{H^+}}} + N_{H_2^+} \sqrt{\frac{kT_e}{m_{H_2^+}}} = N_e \exp \left[\frac{-e(V - V_o)}{kT_e} \right] \left(\frac{kT_e}{2\pi m_e} \right)^{1/2} \quad (1/(\text{cm}^2)(\text{sec})) \quad (10)$$

where N_e is the electron number density and

$$N_e = N_{H^+} + N_{H_2^+} \quad 1/\text{cm}^3 \quad (11)$$

The plasma potential $-V_o$ is measured relative to the point where the positive ions are formed. Rearranging equation (10) gives

$$\exp \left[\frac{-e(V - V_o)}{kT_e} \right] = \sqrt{\frac{2\pi m_e}{m_{H^+}}} \left[\frac{N_{H^+} + N_{H_2^+} \sqrt{\frac{m_{H^+}}{m_{H_2^+}}}}{N_e} \right] \quad (12)$$

Substituting for the various densities gives

$$\exp \frac{-e(V - V_o)}{kT_e} = \frac{\left[X \langle \sigma_{H^+}^{v_e} \rangle_H + (1 - X) \langle \sigma_{H^+}^{v_e} \rangle_{H_2} + (1 - X) \langle \sigma_{H_2^+}^{v_e} \rangle_{H_2} \right] \sqrt{\frac{2\pi m_e}{m_{H^+}}}}{\left[X \langle \sigma_{H^+}^{v_e} \rangle_H + (1 - X) \langle \sigma_{H^+}^{v_e} \rangle_{H_2} + (1 - X) \langle \sigma_{H_2^+}^{v_e} \rangle_{H_2} \right] \sqrt{\frac{m_{H_2^+}}{m_{H^+}}}} \quad (13)$$

Equation (13) can be rewritten as

$$e(V - V_0) = kT_e \ln A \quad (14)$$

where

$$A \equiv -\ln \left\{ \frac{\sqrt{\frac{2\pi m_e}{m_{H^+}}} \left[X \langle \sigma_{H^+ v_e} \rangle_H + (1 - X) \langle \sigma_{H^+ v_e} \rangle_{H_2} + (1 - X) \langle \sigma_{H_2^+ v_e} \rangle_{H_2} \right]}{X \langle \sigma_{H^+ v_e} \rangle_H + (1 - X) \langle \sigma_{H^+ v_e} \rangle_{H_2} + (1 - X) \langle \sigma_{H_2^+ v_e} \rangle_{H_2} \sqrt{\frac{m_{H_2^+}}{m_{H^+}}}} \right\} \quad (\text{eV}) \quad (15)$$

Equation (14) determines the energy that an ion picks up going through the sheath and losses to the wall.

Tonks and Langmuir (ref. 7) consider that the electrons strike the wall with an average kinetic energy of $2kT_e$ per electron, which is the value used herein to account for the sheath effect.

The total power carried to the wall per unit volume of plasma is given by the $1/2 kT_e$ that the ion must have when it reaches the sheath plus the energy gained by an ion going through the sheath (eq. (14)) plus $2kT_e$ given up by an electron times the total number of ions formed. Thus,

$$P_{\text{wall}} = \dot{N}_{\text{ion}} kT_e (2.5 + A) \quad \left(\text{eV}/(\text{cm}^3)(\text{sec}) \right) \quad (16)$$

EFFECT OF DISSOCIATION

All of the power loss mechanisms studied are given as functions of electron temperature. Thus, the percent of power lost by a particular loss mechanism can be determined as a function of electron temperature. The effect of changing the degree of dissociation on the percentage ionization, radiation, and wall loss is studied by dividing the individual loss terms by the total power loss.

DENSITY RATIO

Using equations (6) and (8), one can determine the effect dissociation has on the ratio of the atomic ions produced to the total number of ions produced:

$$\frac{N_{H^+}}{N_{H^+} + N_{H_2^+}} = \frac{X \langle \sigma_{H^+ v_e} \rangle_H + (1 - X) \langle \sigma_{H^+ v_e} \rangle_{H_2}}{X \langle \sigma_{H^+ v_e} \rangle_H + (1 - X) \langle \sigma_{H^+ v_e} \rangle_{H_2} + (1 - X) \langle \sigma_{H_2^+ v_e} \rangle_{H_2} \sqrt{\frac{m_{H_2}}{m_{H^+}}}} \quad (17)$$

RESULTS AND DISCUSSION

The results of the radiation loss term (eq. (3)) are presented in figure 2 for various degrees of dissociation. There is not a significant change in the radiative power loss per unit volume for the various degrees of dissociation in the temperature range considered (3 to 100 eV). The power lost by radiation does, however, vary inversely with the degree of dissociation. Thus, the pure molecular plasma loses more power by ra-

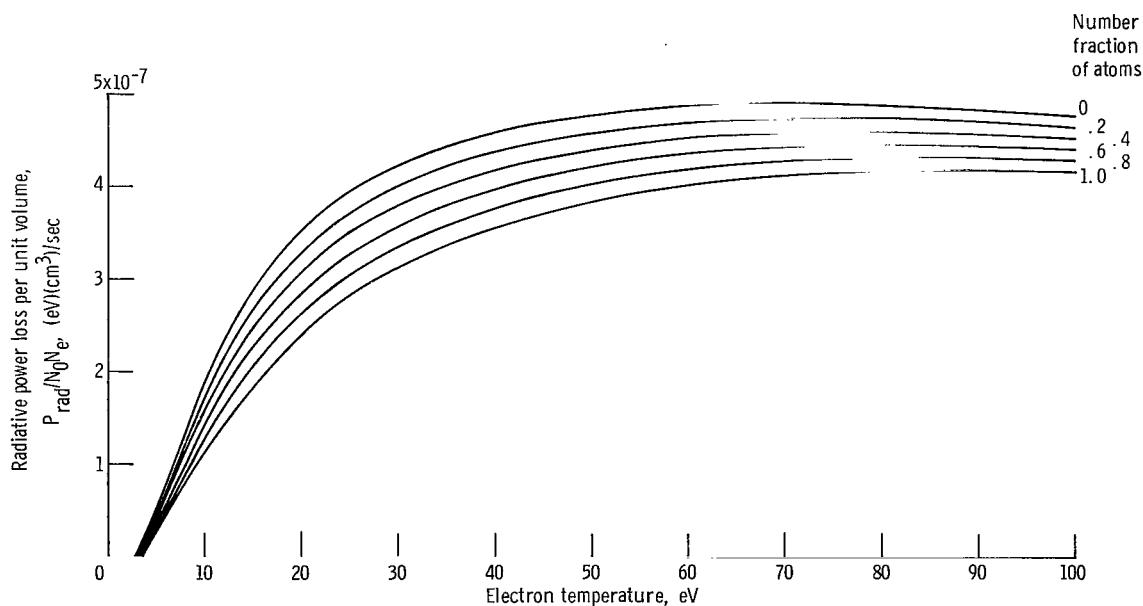


Figure 2. - Power per unit volume lost by radiation for various mixtures of atomic and molecular hydrogen.

diation than the pure atomic plasma. A maximum radiation loss is reached which shifts to higher electron temperatures with increasing degrees of dissociation. The curves are weak functions of electron temperature above 20 electron volts, changing less than a factor of 2 in the temperature range between 20 and 100 electron volts.

The ionization loss term (eq. (2)) is presented in figure 3. As the degree of dissociation decreases, the power lost by ionization increases. Ionization losses do not reach a maximum in the temperature range considered. The difference between the pure atomic and pure molecular ionization loss term is less than a factor of 2. This is true because the ionization cross sections and ionization energies are nearly the same for the atom and the molecule.

The results of the wall loss term (eq. (15)) are given in figure 4. The wall loss term varies inversely with the degree of dissociation. The wall loss term is an order of magnitude larger than either the radiation loss or ionization loss term over much of the temperature range.

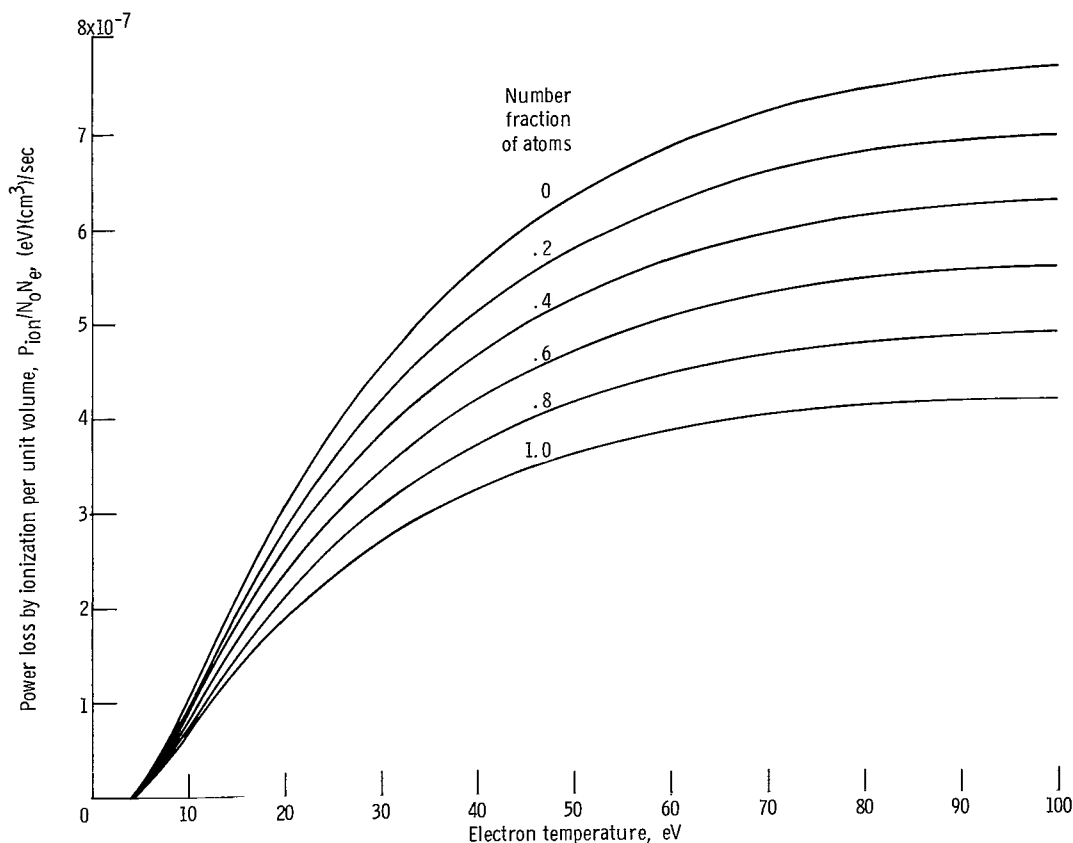


Figure 3. - Power per unit volume lost by ionization for various mixtures of atomic and molecular hydrogen.

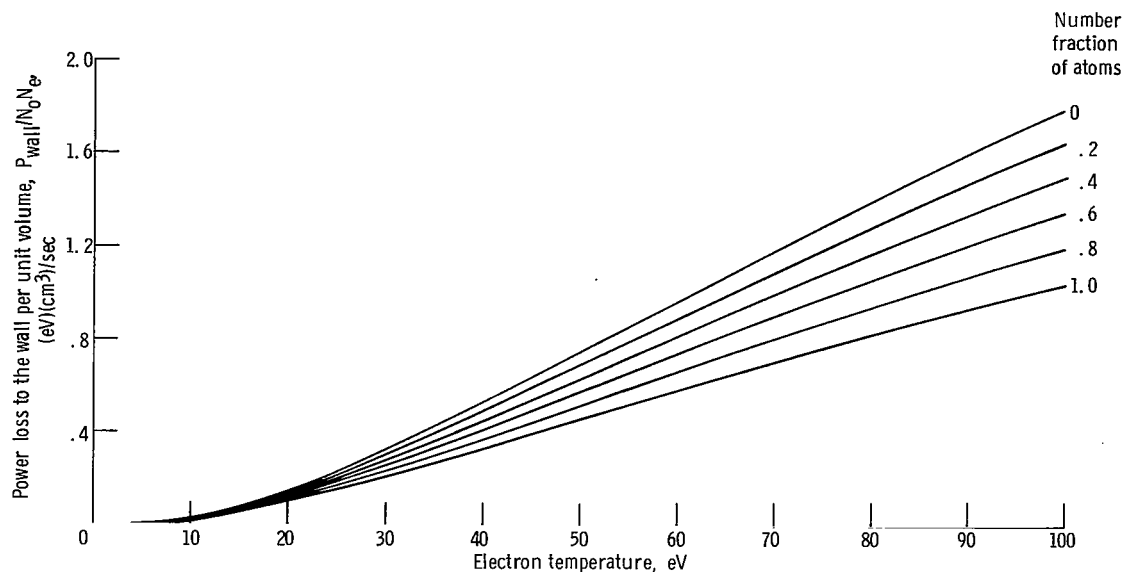


Figure 4. - Power per unit volume lost to wall for various mixtures of atomic and molecular hydrogen.

The individual loss terms are compared in figure 5. They are presented as percentages of the total power loss. The 3 different number fractions of atoms, 0, 0.4, and 1.0, are given in figure 5(a), (b), and (c), respectively. The points at which the wall loss term crosses the other loss terms shift to lower electron temperatures for increasing degrees of dissociation. At electron temperatures below 7 electron volts the radiation loss term predominates. The wall loss term becomes comparable with the radiation loss term for electron temperatures between 7 and 8.5 electron volts for all degrees of dissociation. The peak value for the ionization loss occurs for the pure molecular case and is less than 20 percent of the total power lost. The wall loss term is as much as 94 percent of the total power loss at electron temperatures of 100 electron volts.

Both the radiation and ionization loss terms should be good values because the major cross sections are known experimentally. Even the theoretical cross sections should be correct to within 30 percent over the range considered. This unfortunately is not the case for the wall loss term. The absence of the magnetic field in the theory diminishes the validity of the wall loss term for magnetically confined plasmas. However, for a rough comparison of the wall loss term with volume loss terms, this estimate may suffice.

Figure 6 gives the percentage of active hydrogen ions as a function of electron temperature (determined by eq. (17)). This is done for four different parent gases that range from pure atomic ($X = 1.0$) to 11-percent dissociated ($X = 0.2$). Equation (18) was used for these calculations. Below an electron temperature of 10 electron volts the curves are strong functions of electron temperature. Above 10 electron volts the curves are nearly constant and weakly dependent on electron temperature.

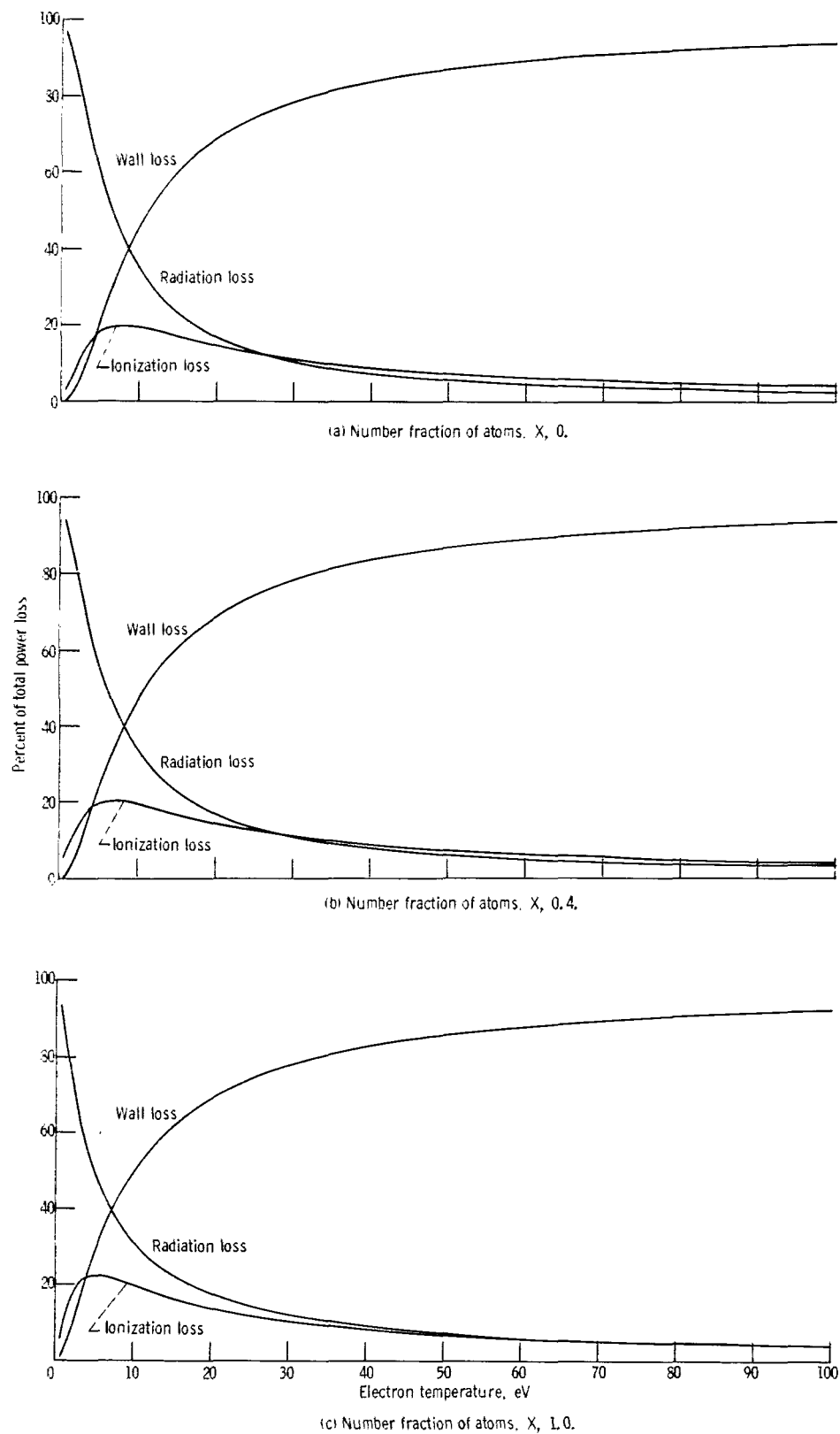


Figure 5. - Power loss terms as percent of total power loss for various values of number fraction of atoms.

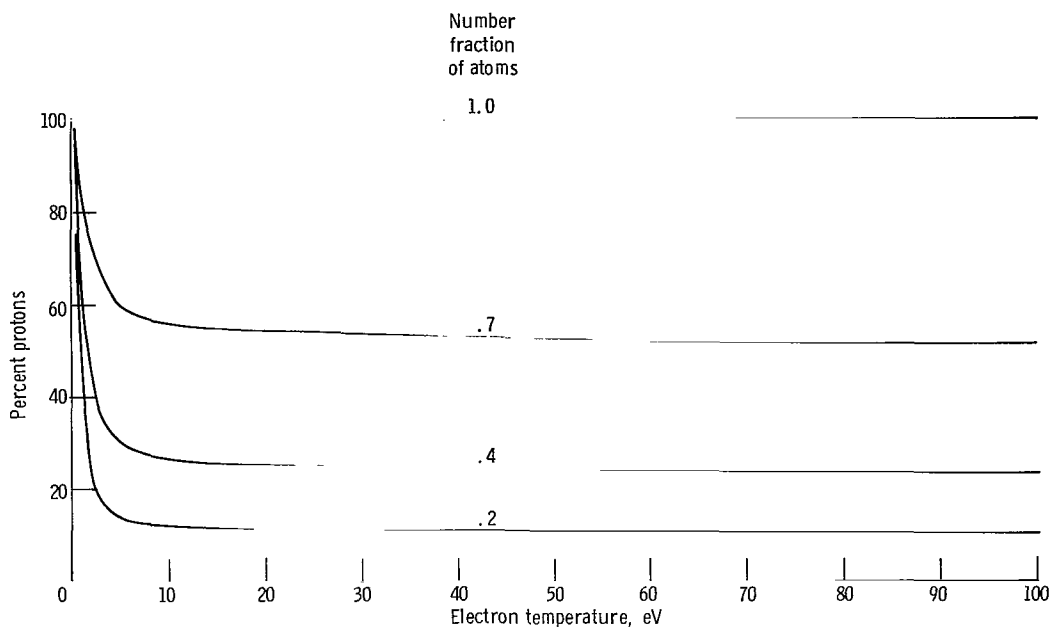
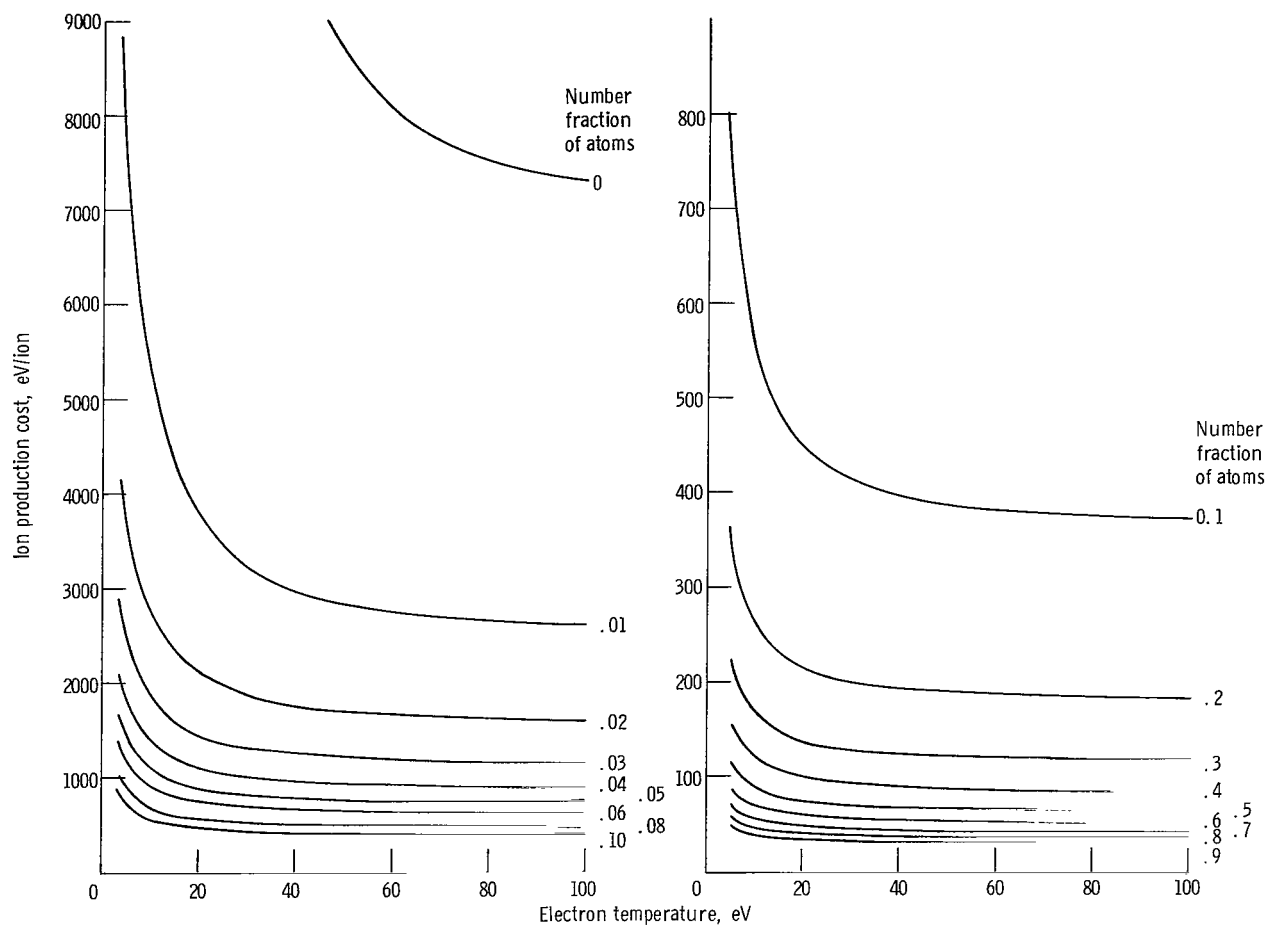


Figure 6. - Percentage of hydrogen ions that are protons for parent gases with various number fractions of atoms.

Figure 7 shows the cost of producing a proton in various mixtures of atomic and molecular hydrogen. Figure 7(a) covers number fractions from 0.0 to 0.1 and figure 7(b) covers the number fractions from 0.1 to 0.9. The change is dramatic. Dissociation of only 0.5 percent ($X = 0.01$) reduces the cost of producing a single proton by a factor of 3 or about 4700 electron volts at an electron temperature of 100 electron volts. Dissociation of 5 percent ($X = 0.1$) reduces the cost by more than an order of magnitude compared with the cost in pure molecular hydrogen. At lower electron temperatures the reduction in cost is even greater. In figure 7(b) it can be seen that high degrees of dissociation do not have as great an effect on reducing the proton cost. Also, the 82-percent dissociation ($X = 0.9$) curve approaches the practical limit for dissociation (ref. 8). Since dissociation cost was not included, no attempt was made to find the optimum degree of dissociation.

Figure 8 gives the ion production term \dot{N}_{ion} as a function of electron temperature. Number fraction of atoms is given as a parameter.



(a) Range of number fraction of atoms, 0.0 to 0.1.

(b) Range of number fraction of atoms, 0.1 to 0.9.

Figure 7. - Proton production cost electron impact of Maxwellian electrons in gas mixture of molecular and atomic hydrogen.

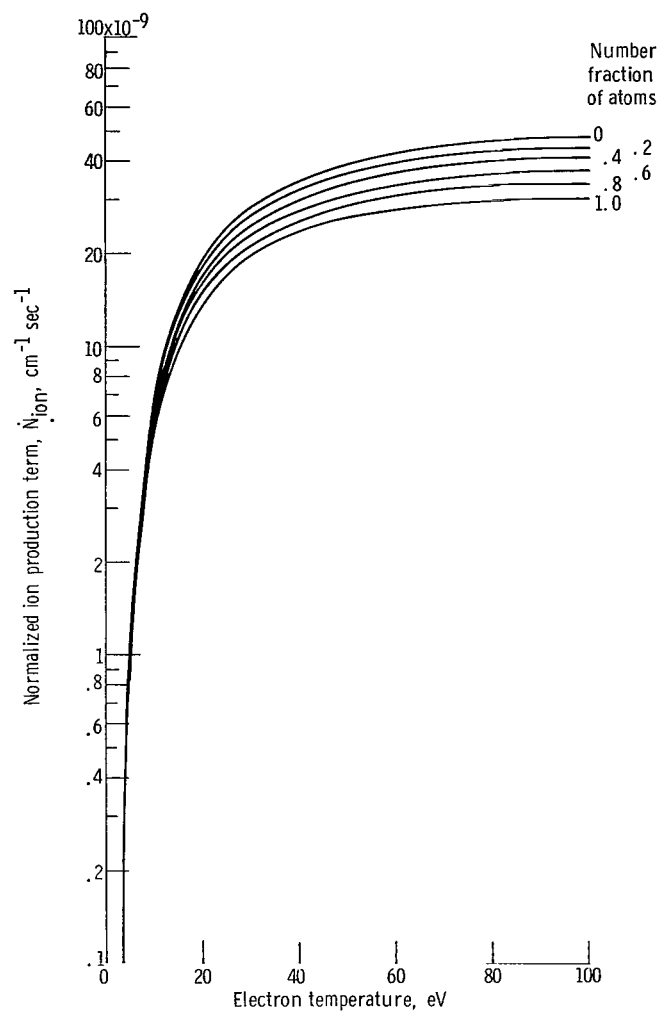


Figure 8. - Ion production term \dot{N}_{ion}/N_0N_e as function of electron temperature.

CONCLUDING REMARKS

Of the three loss mechanisms assumed, the radiation loss and the wall loss pre-dominate. The radiation loss term dominates at low electron temperatures (< 7 eV) and the wall loss term dominates at higher temperatures. The radiation and ionization loss terms are correct since they are based on known cross sections. Further study is needed, however, to arrive at a more precise wall loss term which can include the effect of a magnetic field.

Costs for volume production of protons by electron impact can be greatly reduced if the hydrogen gas is partially dissociated. With 5-percent dissociation ($X = 0.1$), there was more than an order of magnitude decrease in ion production cost over that in molecular hydrogen. At 25-percent dissociation ($X = 0.6$), it takes only three times as much energy to produce a proton as in pure atomic hydrogen.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio May 19, 1969,
129-02-08-02-22.

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